

Consistent Quantum Realism: A Reply to Bassi and Ghirardi

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Received January 26, 2000

A recent claim by Bassi and Ghirardi that the consistent (decoherent) histories approach cannot provide a realistic interpretation of quantum theory is shown to be based upon a misunderstanding of the single-framework rule: they have replaced the correct rule with a principle which directly contradicts it. It is their assumptions, not those of the consistent histories approach, which lead to a logical contradiction.

KEY WORDS: Quantum mechanics; consistent histories; single-framework rule.

1. INTRODUCTION

The first paper on the consistent histories (CH) interpretation of quantum theory was published in the *Journal of Statistical Physics* in 1984.⁽¹⁾ In the years since then this approach, sometimes called “decoherent histories,” has been refined and extended in several books and papers, of which some of the more significant are refs. 2–7. It provides a realistic picture of the atomic realm without the need to invoke quantum measurement as a fundamental principle, and for this reason it can resolve the “measurement problem”⁽⁸⁾ (there are actually two measurement problems, see ref. 9) which has long beset attempts to place the foundations of quantum theory on a sound basis, and which probably cannot be dealt with consistently by traditional methods.⁽¹⁰⁾ Because it resolves various quantum paradoxes^(1, 11) using an analysis based upon the mathematics of Hilbert space, the CH approach removes any need to look for alternatives to standard quantum theory, such as those found in the hidden-variables approach of de Broglie

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and Bohm,⁽¹²⁾ or in the spontaneous reduction ideas of Ghirardi, Rimini, Weber, and Pearle.⁽¹³⁾

There have, to be sure, been a number of criticisms of the CH approach, and these have proven helpful in constructing improved versions of the formalism, and better expositions of its physical interpretation. The most significant of these criticisms are discussed in ref. 6, and reasons are given why they do not invalidate CH quantum theory. This reference provides the material needed to counter various claims, such as in ref. 14, that the CH approach is logically inconsistent or unsound. In this connection it is worth pointing out that even one of its severest critics has admitted that the CH approach is logically consistent when its rules are properly followed.⁽¹⁵⁾

One of these rules, known as the *single framework* (or single family, single logic, or single set) rule, plays a central role in the CH approach, as has been repeatedly emphasized in various publications.⁽⁴⁻⁷⁾ Despite the extensive discussion of this rule in the CH literature, accompanied by numerous applications to specific problems, it is still sometimes misunderstood, as in some recent work by Bassi and Ghirardi.⁽¹⁶⁻¹⁸⁾ In particular, these authors have claimed, in an article⁽¹⁷⁾ appearing in this Journal, that the CH interpretation of quantum theory when interpreted in a realistic way using some reasonable assumptions leads to contradictions in the sense of violating a result of Bell,⁽¹⁹⁾ and Kochen and Specker.⁽²⁰⁾ On the face of it this seems rather surprising. The Bell-Kochen-Specker result shows that a certain type of hidden-variables approach to quantum theory can lead to a contradiction because it makes assumptions incompatible with the structure of Hilbert space. On the other hand, the CH interpretation has been explicitly constructed to take account of the structure of Hilbert space, and does not rely on hidden variables in any way.

Closer examination shows that the Bassi and Ghirardi argument violates the single-framework rule, and thus the claimed contradiction with Bell-Kochen-Specker is not a consequence of the principles of the CH approach, but is instead due to Bassi and Ghirardi's having rejected those principles. This was pointed out in ref. 21 in response to ref. 16, but since ref. 17 is considerably longer and also somewhat clearer than ref. 16, raises the issue in a some-what different way, and has appeared in a different journal, a separate reply to it seems appropriate. The present article, in order to be self-contained, contains a certain amount of overlap with ref. 21.

Since the arguments in ref. 17 (as in ref. 16) deal entirely with the Hilbert space description of a system at a single time, most of the formal machinery of the CH approach—histories, consistency, and assignment of probabilities by use of the time-dependent Schrödinger equation—is not

needed for the following discussion. The essential point we wish to make is that a quantum Hilbert space differs in crucial respects from a classical phase space, and this mathematical difference must be reflected in any valid physical interpretation of quantum theory. Importing “intuitively obvious” ideas of classical physics into quantum mechanics without paying adequate attention to the mathematical structure of the latter, in direct contradiction to the rules of the CH interpretation, is what has given rise to the contradiction noted by Bassi and Ghirardi, as we shall show.

Before dealing with the main issue, we need to indicate the connection between truth functionals—our name for the homomorphisms denoted by h in ref. 17—and certain elementary concepts from standard probability theory. This is done in Section 2, and the quantum counterparts of truth functionals and probability concepts are taken up in Section 3, along with the single-framework rule. In Section 4 we show that in their argument Bassi and Ghirardi have mistakenly replaced the single-framework rule with what we call the *every-framework principle*, which is not only not the same as the single-framework rule, but stands in direct contradiction to it; for this reason their argument has basically nothing to do with CH quantum theory. Section 5 responds to some other less-important issues in ref. 17, and Section 6 has a brief conclusion.

2. SAMPLE SPACES AND TRUTH FUNCTIONALS

A basic concept in elementary probability theory is that of a *sample space*. According to Feller,⁽²²⁾ the possible outcomes of an idealized experiment correspond to precisely one and only one point of the sample space. If a coin is tossed, the sample space consists of two possibilities, H and T; if a die is rolled, there are six points in the sample space. Before the idealized experiment is carried out one does not, in general, know what the outcome will be, but when it has taken place, *one and only one* of the outcomes actually occurs, i.e., is the *true* result of the experiment. The books on probability theory with which I am familiar do not seem to employ the terms “true” and “false,” but the way in which they define a sample space justifies the association of “true” with the sample point that represents the actual outcome, and “false” with all the others. In addition, Feller distinguishes *simple* events, the elements of the sample space, from *compound* events which are associated with some subset of the elements of the sample space. A compound event is “true” if it contains the point of the sample space which actually occurs, and is “false” otherwise.

Rather than the terminology of ordinary probability theory, ref. 17 uses what I call a *truth functional*: a homomorphism (there denoted by h) from a Boolean algebra of events to the set $\{0, 1\}$, also thought of as a

Boolean algebra. In light of the preceding remarks, the connection of a sample space, and its corresponding event algebra, to a system of truth functionals can be explained in the following way. Let \mathcal{S} be the sample space, and \mathcal{P} be some subset of \mathcal{S} , thus a compound event in Feller's terminology. The *indicator* P of \mathcal{P} is a function on \mathcal{S} taking the value 1 at all points which lie in \mathcal{P} and 0 at all other points of \mathcal{S} . The usual Boolean algebra of subsets of \mathcal{S} is then isomorphic to a Boolean algebra \mathcal{B} of indicator functions in which the greatest element is the function I , equal to 1 at all points of \mathcal{S} ; the least element is 0, equal to 0 everywhere; the complement of an indicator P is $I - P$; the join $P \cap Q$ of two indicators is their product PQ ; and the meet $P \cup Q$ is the indicator $P + Q - PQ$.

A truth functional θ is then a function which assigns to each indicator in the algebra \mathcal{B} either the value 1 (true) or 0 (false) in a way which satisfies the following three conditions:

$$\theta(I) = 1, \quad \theta(I - P) = 1 - \theta(P), \quad \theta(PQ) = \theta(P) \theta(Q) \quad (1)$$

It is not hard to show that any such function is necessarily of the form

$$\theta_q(P) = P(q) = \begin{cases} 1 & \text{if } q \in \mathcal{P} \\ 0 & \text{if } q \notin \mathcal{P} \end{cases} \quad (2)$$

where q is some point in the sample space \mathcal{S} . One should think of θ_q as the truth functional appropriate for the case in which the sample point q actually occurs, or is true, since it then assigns the value 1 (true) to every compound event which contains q , and 0 (false) to the ones which do not contain q . If the sample space is discrete, one can think of $\theta_q(P)$ in probabilistic terms as the conditional probability of P given q , assuming the probability of q is greater than zero, so that the conditional probability is defined. It is in this sense, among others, that one can say that "true" is associated with a (conditional) probability of 1, and "false" with probability 0, in a probabilistic theory.

This approach can be employed in classical statistical mechanics in the following way. Let γ be a representative point of the phase space Γ . A *physical property* P of the system corresponds to the subset \mathcal{P} of Γ consisting of those points γ for which this property is true. The corresponding indicator $P(\gamma)$ is 1 whenever γ is in \mathcal{P} , and 0 otherwise. For example, if P is the property that the total energy of a one-dimensional harmonic oscillator is less than some constant E_0 , \mathcal{P} is the region inside an appropriate ellipse in the x, p plane (x the position, p the momentum), and $P(\gamma)$ is 1 for γ inside and 0 for γ outside this ellipse.

Now consider a coarse graining of the phase space into a collection \mathcal{D} of N non-overlapping regions or “cells.” We can write the identity indicator I (equal to 1 for all γ) in the form

$$I = \sum_{j=1}^N D_j \tag{3}$$

where D_j is the indicator corresponding to the j th cell. Since the cells do not overlap it follows that

$$D_j D_k = \delta_{jk} D_j \tag{4}$$

consistent with the obvious fact that $I^2 = I$. The set of 2^N indicators which can be written as

$$P = \sum_{j=1}^N \pi_j D_j \tag{5}$$

with π_j is either 0 or 1, form a Boolean algebra \mathcal{B} using the definitions of complement, meet, and join introduced earlier. A truth functional θ is a function on \mathcal{B} taking the values 0 or 1 in a way which satisfies (1), so it has the form

$$\theta_k(P) = \begin{cases} 1 & \text{if } PD_k = D_k \\ 0 & \text{if } PD_k = 0 \end{cases} \tag{6}$$

where D_k is one of the elements of (3). Note that the collection \mathcal{D} of cells constitutes a sample space, because in any given “experiment” the phase point γ representing the system will be in one and only one of the cells. The truth functional θ_k corresponds to the case in which the phase point γ is somewhere in the cell \mathcal{D}_k ; it assigns the value 1 to all collections of cells whose union contains the phase point, and 0 to all others. One can again interpret $\theta_k(P)$ as a conditional probability, assuming that the probability assigned to \mathcal{D}_k is positive.

Notice that it is because we are assuming that P is of the form (5) that the product $P D_k$ must have one of the two forms on the right side of (6): no property of the form (5) can include part but not all of some cell D_k . Consequently, (6) defines a truth functional for indicators belonging to this particular algebra \mathcal{B} , but not for all possible properties; in this sense a truth functional is relative to a particular coarse graining \mathcal{D} , or its Boolean algebra \mathcal{B} . However, in classical mechanics it is possible to construct a *universal truth functional* which is not limited to a single Boolean algebra, but which will assign 0 or 1 to *any* indicator on the classical phase space

in a manner which satisfies (1). To do this, choose some point γ_q in Γ , and let

$$\theta_q(P) = P(\gamma_q) \quad (7)$$

That is, θ_q assigns the value 1 to any property which contains the point γ_q , and 0 to any property which does not contain this point, in agreement with how one would normally understand “true” in a case in which the state of the system is correctly described by the phase point γ_q .

3. QUANTUM TRUTH FUNCTIONALS AND THE SINGLE FAMILY RULE

The quantum counterpart of a classical phase space is a Hilbert space \mathcal{H} . For our purposes it suffices to consider cases in which \mathcal{H} is of finite dimension, thus avoiding the mathematical complications of infinite-dimensional spaces. Following von Neumann,⁽²³⁾ we associate a quantum property, the counterpart of a set of points in the classical phase space, with a linear subspace \mathcal{P} of \mathcal{H} , or the corresponding orthogonal projection operator or *projector* P onto this subspace. If I is the identity operator, the negation of a property P corresponds to the projector $I - P$, and the conjunction $P \wedge Q$ of two properties corresponds to the projector PQ in the case in which P and Q commute with each other. If $PQ \neq QP$, then neither PQ nor QP is a projector, so there is no obvious way to define a property corresponding to the conjunction, an issue to which we shall return.

The quantum counterpart of a coarse graining of a classical phase space is a *decomposition of the identity*, a collection of mutually orthogonal projectors $\{D_j\}$ satisfying (4) whose sum is the identity, as in (3). This decomposition gives rise to a set of projectors of the form (5), all of which commute with each other, and which form a Boolean algebra \mathcal{B} analogous to the algebra of classical indicator functions. One can define a quantum truth functional θ on the elements of \mathcal{B} in the manner indicated previously: it assigns to every projector P in \mathcal{B} a value 0 or 1 in a way which satisfies the three conditions in (1). Once again, any truth functional of this type can be written in the form (6) for some k , and thus there is a one-to-one correspondence between truth functionals and the elements of \mathcal{D} , which one can think of as the quantum version of a sample space.

The CH approach to quantum theory is “realistic” in the sense that it treats the members of a particular decomposition of the identity, a quantum sample space, as mutually exclusive possibilities, one and only one of which occurs, or is true, for a particular physical system at a particular instant of time, in precisely the same sense as in classical statistical

mechanics. The difference between quantum and classical physics emerges not when one considers a single quantum sample space, but when one asks about the relationship between two or more *different* sample spaces. Here quantum theory is very different from classical physics because the product of two quantum projectors P and Q on the same Hilbert space can depend upon the order, and when PQ is unequal to QP , neither of these products is a projector. By contrast, the product of two indicators on the same classical phase space is always an indicator, since multiplication is commutative. For example, for a spin-half particle with components of angular momentum S_x , S_y , and S_z (in units of \hbar), the projector for the property $S_x = +1/2$, which is $\frac{1}{2}I + S_x$, does not commute with $\frac{1}{2}I + S_z$, the projector for $S_z = +1/2$. Consequently, a key question in quantum theory, with no counterpart in classical physics, is: How can one make sense out of the conjunction of two quantum properties, such as $S_x = +1/2$ AND $S_z = +1/2$, when the corresponding projectors do not commute with each other?

The answer of the consistent historian is that one *cannot* make sense of $S_x = +1/2$ AND $S_z = +1/2$; it is a *meaningless* statement in the sense that (CH) quantum theory assigns it no meaning. There are no hidden variables, and thus there is a one-to-one correspondence between quantum properties and subspaces of the Hilbert space in CH quantum theory. Since every one-dimensional subspace of the two-dimensional Hilbert space \mathcal{H} of a spin-half particle corresponds to a spin in a particular direction, there is no subspace left over which could plausibly represent the property $S_x = +1/2$ AND $S_z = +1/2$. To be sure, one might assign to it the zero element of \mathcal{H} , a zero-dimensional subspace corresponding to the property which is always false (analogous to the classical indicator which is 0 everywhere). This, in fact, was the proposal, for this particular situation, of Birkhoff and von Neumann in their discussion of quantum logic.⁽²⁴⁾ It is important to notice the difference between their approach and the one used in CH. A proposition which is meaningful but false is very different from a meaningless proposition: the negation of a false proposition is a true proposition, whereas the negation of a meaningless proposition is equally meaningless. The Birkhoff and von Neumann approach requires, as they themselves pointed out, a modification of the ordinary rules of propositional logic, whereas the CH approach does not.² However, in CH quantum theory it then becomes necessary to exclude meaningless talk from meaningful discussions, a task which is not altogether trivial.

Generalizing from this example, the CH approach requires that a meaningful probabilistic description of a single quantum system at a particular time must employ a *single framework*: a single Boolean algebra

² For further remarks on some of these issues, see Section IV A of ref. 6.

of commuting projectors generated, in the sense of (5), from a specific decomposition of the identity or quantum sample space. To be sure, *many alternative descriptions* can be constructed using different decompositions of the identity: the single-framework rule is certainly not intended to restrain the imagination of theoretical physicists! However, *combining* results from *different sample spaces* into a single description is forbidden by the single-framework rule, apart from the following exception.

Two frameworks involving properties of a single system at a single time (we are ignoring genuine histories, for which the rules are more complex) are said to be *compatible* provided the two Boolean algebras are parts of a single, larger Boolean algebra of commuting projectors. This is true if and only if every projector belonging to one of the original algebras commutes with every projector belonging to the other algebra, which in turn is the same as requiring that the projectors from the two decompositions of the identity, or sample spaces, commute with one another. A larger collection of frameworks is compatible if all pairs are compatible, and frameworks are said to be mutually *incompatible* if they are not compatible. Descriptions based upon two or more compatible frameworks can always be combined by using the single Boolean algebra which contains all of the different (mutually commuting) algebras, and thus one is still employing a single framework, corresponding to a single decomposition of the identity, in accordance with the single-framework rule.

The single-framework rule is not at all unreasonable from the perspective of elementary probability theory, where problems are generally set up using a single sample space. Thus if a coin is to be tossed ten times in a row, the statistical properties are worked out not by constructing ten sample spaces, but by using a single sample space containing 2^{10} points. The single-framework rule is also perfectly compatible with classical statistical mechanics, for if one uses two or more coarse grainings of the phase space, the results can always be combined by means of a single coarse graining which uses a collection of cells generated by intersections of other cells in an obvious way. Thus ordinary probabilistic arguments and classical statistical mechanics satisfy the single-framework rule, albeit in a somewhat trivial sense.

As already noted, the single-framework rule as applied to Boolean algebras of properties refers to a *single system* at a *single instant* of time. Given two nominally identical systems, there is no reason why one cannot use one framework for the first and a different framework for the second. For instance, in the case of two spin-half particles, $S_x = +1/2$ could be a correct description of one of them at the same time that $S_z = +1/2$ is a correct description of the other. Similarly, the same particle may have $S_x = +1/2$ at an earlier and $S_z = +1/2$ at a later time. Conversely, when

incompatible frameworks turn up in some discussion of a quantum system, it is best to think of them as referring to different systems, or to a single system at different times, or perhaps simply as tentative or hypothetical proposals without any suggestion that they should be taken in a realistic sense. (Ascribing properties to a single system at more than one time requires the use of a history, and this requires additional considerations which lie outside the scope of the present discussion.)

In any application of probability theory, precisely one of the elements of the sample space is thought of as existing, or “true” in any realization of an ideal experiment. In this sense the notion of “truth” in a probabilistic theory is necessarily connected with, and thus depends upon the sample space or framework one is considering. In classical physics one can forget about this dependence, because if more than one framework is under consideration, in the case of a single system at a single time, they can always be combined into a single framework. This is reflected in the fact that one can always define a universal truth functional for a classical phase space, as noted in Section 2. Because of the possibility that frameworks can be incompatible, the framework dependence of “true” is not at all trivial in quantum physics; indeed, one might say that this is one of the main ways in which the mathematical structure of quantum theory forces one to adopt a different kind of physical interpretation from what one is used to in classical mechanics.

In particular, in quantum theory *there is no universal truth functional* θ_q which can be used to assign values of 0 and 1 to all projectors in a way which agrees with the three conditions in (1). In a certain sense this is immediately obvious for any Hilbert space of dimension 2 or more, since in such a space there will always be projectors P and Q which do not commute with each other. In such a case PQ is not a projector, and the third condition in (1) is not even defined, much less satisfied. One might hope to get around this problem by modifying the third condition and only requiring that it hold in cases in which P and Q commute with each other. This, however, gains very little, for the results of Bell and of Kochen and Specker referred to earlier demonstrate that even such a “modified” universal truth functional does not exist for a Hilbert space of dimension 3 or more. (A simple example due to Mermin, showing the impossibility of a universal truth functional in a Hilbert space of dimension 4, is discussed in ref. 21.)

The absence of a universal truth functional causes no difficulties for CH quantum theory because of the single-framework rule, which prevents the comparison of incompatible frameworks. The situation is different for the alternative principle proposed by Bassi and Ghirardi, which they have somehow managed to confuse with the single-framework rule, and which will be taken up next.

4. THE EVERY-FRAMEWORK PRINCIPLE OF BASSI AND GHIRARDI

In ref. 17 Bassi and Ghirardi introduce, in the discussion leading up to and including their (6.1), what I shall call the “every-framework principle,” which in the notation of the present paper can be stated in the following way.

Consider a quantum Hilbert space, and let $\{\mathcal{D}_f\}$ be the different possible decompositions of the identity, where f is a label which takes on uncountably many values. For example, for a spin-half particle, f will run over all directions in space w , as long as $+w$ is identified with $-w$, since each decomposition of the identity corresponds to a sample space with just the two points $S_w = \pm 1/2$. Corresponding to \mathcal{D}_f there is a corresponding Boolean algebra \mathcal{B}_f of projectors of the form (5). Given a projector P , we define

$$\mathcal{F}(P) = \{f : P \in \mathcal{B}_f\} \quad (8)$$

to be the collection of labels such that P is a member of the Boolean algebra \mathcal{B}_f .

The *every-framework principle* asserts that there is a collection of truth functionals $\{\theta_f\}$, one for each decomposition of the identity, with the following property: if P is any projector in the Hilbert space, *the value of $\theta_f(P)$ is the same for all f in $\mathcal{F}(P)$* . That is to say, P is assigned precisely the same truth value, 0 or 1, by all members of the collection $\{\theta_f\}$ for which $\theta_f(P)$ is actually defined.

The every-framework principle has a certain intuitive appeal when one is thinking of a single system at a single time. It is actually correct for classical statistical mechanics, where a property P is true as long as the representative phase point γ is inside the corresponding set \mathcal{P} , and false otherwise. Hence to construct a collection of truth functionals, associated with a collection of coarse grainings, satisfying the every-framework principle, one simply chooses some representative point γ_q in the phase space, and for a coarse graining \mathcal{D}_f lets the indicator for the cell which contains γ_q play the role of the special D_k in (6). Or, to put the matter in a slightly different way, one simply lets θ_f be the restriction to \mathcal{B}_f of the universal truth functional defined in (7).

Given this result, one is not surprised to learn that the quantum-mechanical version of the every-framework principle implies the existence of a universal truth functional θ_u of the modified form discussed towards the end of Section 3. For each P , one sets $\theta_u(P)$ equal to the common value specified by the every-framework principle, noting that every P is contained

in at least one decomposition of the identity, that consisting of P and $I - P$. Since when restricted to the Boolean algebra \mathcal{B}_f the functional θ_u is identical to θ_f , it is at once evident that the first two requirements in (1) will always be satisfied, while the third will be satisfied in cases in which P and Q commute, since there is then at least one Boolean algebra \mathcal{B}_g which contains both of them, and θ_u when restricted to \mathcal{B}_g is the same as the corresponding truth functional θ_g .

But, as we have already noted, the *nonexistence* of a universal truth functional has been proven mathematically for any Hilbert space of dimension greater than two. Consequently, *the every-framework principle is in clear contradiction with the principles of quantum mechanics*. The proofs of this fact given in refs. 16 and 17 are correct but superfluous; they simply repeat what is already well known to people who work in the foundations of quantum theory.

What is the relationship of the every-framework principle and the single-framework rule? They are mutually contradictory, for fairly obvious reasons. The every-framework principle requires us to compare the results of truth functionals, or equivalently sample spaces, associated with different and in general incompatible frameworks, in precisely the manner forbidden by the single-framework rule. According to the latter, such a comparison makes no sense in the case of incompatible frameworks. (Note that it is only with the help of incompatible frameworks that one can reach a contradiction using the Bell–Kochen–Specker approach, so incompatible frameworks are essential to the argument in ref. 17.) To be sure, two different frameworks might refer to two different systems (or the same system at two different times), but in that case there is no reason whatsoever to expect that a particular property has the same truth value for the two systems, and thus no motivation for invoking the every-framework principle.

One must admit that the every-framework principle has a certain intuitive appeal: how could the truth value of some physical property possible depend upon the sample space in which it is embedded? Surely if it is true it is really true, apart from anything one can say about the rest of the world, and if it is false it is false! This appeal is seductive because it focuses attention on the *physical property* rather than on the *sample space*. When, however, one pays attention to the latter, things appear in a quite different light. Let us consider as an example two *incompatible* quantum sample spaces

$$\mathcal{S}_1 = \{A, B, C\} \quad \mathcal{S}_2 = \{A, D, E\} \quad (9)$$

where A , B , and C are three projectors which add up to I , and likewise $A + D + E = I$. However, neither B nor C commutes with either D or E .

Let us employ the every-framework principle, and suppose that A is *false* in both \mathcal{S}_1 and \mathcal{S}_2 . Because \mathcal{S}_1 is a sample space, this means that either B or C is true, and because \mathcal{S}_2 is a sample space, either D or E must be true. Suppose for the sake of argument that it is B which is true in \mathcal{S}_1 and D which is true in \mathcal{S}_2 . Then if we insist that \mathcal{S}_1 and \mathcal{S}_2 apply to the same system at the same time, this means that two properties represented by *non-commuting projectors* are simultaneously true. For example, they could be $S_x = +1/2$ and $S_z = +1/2$ for a spin-half particle. This is hard to reconcile with the Hilbert space structure of ordinary quantum mechanics, as pointed out in Section 3, and in CH quantum theory it is forbidden by the single-framework rule. Thus we see that when A is false, the every-framework principle has certain implications which, when brought to light, make it much less appealing.

The case in which A is *true* in both \mathcal{S}_1 and \mathcal{S}_2 also leads to unsatisfactory results. Because \mathcal{S}_1 and \mathcal{S}_2 are sample spaces, the truth of A means that all four properties B , C , D , and E are false. One might be tempted to suppose that the falsity of two incompatible properties is unproblematical—after all, who cares about things which do not occur? The trouble is that when B is false, its negation $\tilde{B} = I - B$ is true. Furthermore, if two projectors B and E do not commute, the same is true of their negations \tilde{B} and $\tilde{E} = I - E$. Thus using the every-framework principle once again leads to the conclusion that two properties represented by non-commuting operators are simultaneously true. In summary, whatever may be its initial intuitive appeal, much of the allure of the every-framework principle vanishes when one realizes what it really means.

Let us look at this example from a slightly different perspective, by introducing a third sample space

$$\mathcal{S}_0 = \{A, \tilde{A} = I - A\} \quad (10)$$

containing only A and its negation. This is obviously the smallest sample space in which “ A is true” and “ A is false” make sense. The sample space \mathcal{S}_0 is compatible with both \mathcal{S}_1 and with \mathcal{S}_2 , each of which represents a *refinement* of \mathcal{S}_0 . The rules of quantum reasoning employed in ref. 5 allow one to deduce that if A is true/false in \mathcal{S}_0 , then it is also true/false in \mathcal{S}_1 , and the same deduction is possible going from \mathcal{S}_0 to \mathcal{S}_2 . However, the single-framework rule prevents combining these results in a single description: one cannot employ both \mathcal{S}_1 and \mathcal{S}_2 for the same system at the same time, for the reasons indicated previously. This means that at least some of the intuitive appeal which seems to lie behind the every-framework principle, the notion that the truth of some property should not depend upon what else is going on in the world, is supported in CH quantum theory.

And this can be done in a consistent way without leading to any logical contradictions precisely because the CH approach employs the single-framework rule rather than the every-framework principle.

The single-framework rule and the every-framework principle are, thus, completely incompatible with each other, whether one regards them either from a purely formal perspective—the former forbids combinations which the latter allows—or in terms of their intuitive significance. Hence one can only regard with astonishment the claim of Bassi and Ghirardi, found in the very next sentence after their (6.1), that the every-framework principle constitutes the “only reasonable way” to interpret the single-framework rule! It is hard to imagine a more serious misunderstanding of a rule that has been stated over and over again in the literature on CH quantum theory, and illustrated by means of numerous examples. Discussions and criticisms of the single-framework rule can be a valuable component of the scientific enterprise. But to introduce a new principle which is not only different from, but directly contrary to the single-framework rule, and then claim that the former is the only reasonable way to interpret the latter does nothing but cause confusion.

5. SOME OTHER ISSUES

Aside from the every-framework principle, there are some other points in ref. 17 (and also ref. 18) which merit at least a brief response.

- In Sections 4 and 9 of ref. 17, Bassi and Ghirardi assert that what I call MQS (macroscopic quantum superposition) states—for example, Schrödinger’s infamous cat—are physically unacceptable, and fault the CH approach for not providing some criterion for excluding them.

In response, it will help to use an analogy from classical physics, while remembering that any classical analogy can only go part way in helping us understand quantum phenomena. A coin spontaneously rising a centimeter above a table on which it is sitting at rest is physically unacceptable in the sense that such a violation of the second law of thermodynamics is never observed to occur, despite the fact that nothing in the laws of classical mechanics excludes such a possibility. We understand why we never observe such things by using statistical mechanics, which assigns an extremely small probability to such an event. That is, we have a scientific understanding of why violations of the second law are not observed, despite the fact that the laws of classical (and also quantum) mechanics permit such possibilities.

The quantum Hilbert space certainly contains MQS states, because it is, by definition, a linear vector space—which includes superpositions of any of its elements. However, MQS states are incompatible, in the quantum

sense, with the quasi-classical framework(s)⁽³⁾ needed to describe our ordinary experience with macroscopic objects. Thus the single-framework rule tells us that it makes no sense trying to include MQS states in descriptions of the everyday world of human experience. Conversely, if a quantum description employs the MQS state that is (formally) a linear superposition of a live and dead cat, it makes no sense, according to the single-framework rule, to think of the whatever-it-is as somehow involving a cat, for the properties typically used to identify a cat will, in quantum theory, be represented by projectors which do not commute with the MQS state, and are therefore of no use for discussing the meaning of such a state. In this sense, at least, CH quantum theory does provide criteria for excluding MQS states from certain types of quantum descriptions. Quantum physicists who refuse to employ the single-framework rule must, of course, find some other means of disposing of, or perhaps peacefully coexisting with MQS states. It is also worth noting that the reason quantum superpositions states of this sort cannot be detected in the laboratory, even for microscopic objects, as long as they contain a substantial number of atoms, is by now reasonably well understood in terms of the process of decoherence, a topic which has been treated from the CH perspective by Omnès.⁽⁷⁾ (Decoherence is much like classical irreversibility, making the jumping coin an even better analogy.) To summarize the situation, CH quantum theory certainly permits descriptions using MQS states, but at the same time provides an explanation as to why they are neither needed nor particularly useful for a science of the macroscopic world.

- In a not unrelated point, Bassi and Ghirardi suggest that one may be able to get around the difficulties they have encountered by employing their every-framework rule by making a drastic reduction in the set of consistent families which can be considered to be physically significant.

In response, there is nothing wrong with these authors announcing a direction for their future research, as long as they make it plain that the motivation for it comes not from any problem involving the CH approach, but rather from the disagreement between their every-framework principle (itself completely contrary to the CH single-framework rule) and standard quantum theory. No proposal for using a restricted class of families in the manner they propose has thus far turned out to be very useful for quantum interpretation, but no doubt those who consider this a worthwhile approach will continue the search.

- Bassi and Ghirardi take the position, both in Section 6 of ref. 17 and in ref. 18, that the only alternative to their every-framework principle in which any property P has the precisely the same truth value in every framework which contains it, is to suppose that in certain of frameworks

it is true and in other frameworks it is false, something they consider unacceptable.

The response to this is contained in the material in Section 4 above, but it may be worthwhile making it quite explicit. From the CH perspective, using two incompatible frameworks to describe the same system at the same time is not meaningful—this is precisely the point of the single-framework rule. Since meaningless truth values are meaningless, there is no reason to be concerned about whether they agree or disagree. Alternatively, one can suppose that two incompatible frameworks do *not* refer to the same system at the same time. In that case, there is no a priori reason to expect the truth values for a particular property to be the same, and so no reason to be worried if they are different.

- In ref. 18 Bassi and Ghirardi assert that in the previous literature on consistent histories the single-framework rule was not explained well enough or clearly enough so as to obviously exclude their every-framework principle.

It is quite true that the language of truth functionals was not employed by consistent historians (so far as I am aware) prior to the recent ref. 21. Previous work used the standard language of elementary probability theory, with its sample spaces and event algebras, and assumed the usual association between probability theory and reality, as pointed out, for example, in Section 7.2.3 of ref. 1. A basic understanding of how sample spaces function in ordinary probability theory is all that one really needs in order to understand the CH approach and the significance of the single-framework rule, including the fact that it is quite contrary to the every-framework principle. The use of truth functionals, while it may be advantageous for some purposes, is not actually needed.

- At the beginning of Section 7.1 in ref. 17, Bassi and Ghirardi, in a footnote, issue a challenge to me and an anonymous referee to identify which of their four precisely formulated (in their opinion) assumptions are inconsistent with the CH single-framework rule, and accept the consequences of this identification.

In fact these four assumptions are not precisely formulated, as was pointed out in ref. 21. Writing in response to that, Bassi and Ghirardi⁽¹⁸⁾ have themselves identified their assumption (c) as the one which is incompatible with the single-framework rule, and I see no reason to dispute this. That rejecting their (c)—the every-framework principle—leads to dire consequences is not true, as should be clear from the discussion in Section 4 above. Instead, it allows a sensible discussion of quantum properties using consistent quantum principles.

6. CONCLUSION

In ref. 17 Bassi and Ghirardi have, in essence, substituted their every-framework principle for the single-framework rule of CH quantum theory, and then concluded, correctly, that the every-framework principle makes no sense in the quantum world. Their only mistake is in supposing that the every-framework principle has something to do with CH quantum theory, whereas in fact the two are directly contrary to each other. While this error is easily spotted by someone who is familiar with CH methods, it is nonetheless regrettable that others less familiar with them have, once again, been given the mistaken impression that there is something logically unsound, or at least suspicious, about CH quantum theory.

To be sure, Bassi and Ghirardi and other critics of CH perform a valuable function in looking for flaws in this approach. Their failure (at least thus far) to find anything wrong with CH, while at the same time demonstrating that the various alternatives that they propose possess serious flaws, adds to one's confidence that the CH approach does, in fact, provide a satisfactory realistic interpretation of quantum theory.

ACKNOWLEDGMENTS

The author is indebted to T. A. Brun and O. Cohen for reading and commenting on the manuscript. The research described here was supported by the National Science Foundation Grant PHY 99-00755.

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